Basic Scheduling Algorithms for Single Machine Problems: Due-Date Scheduling

1. Minimising maximum lateness

In this section we develop an algorithm that finds an optimal schedule for problem $1||L_{\max}$, where

$$L_{\max} = \max\{L_j = C_j - d_j \mid j=1,\ldots, n\}$$

is the maximum lateness.

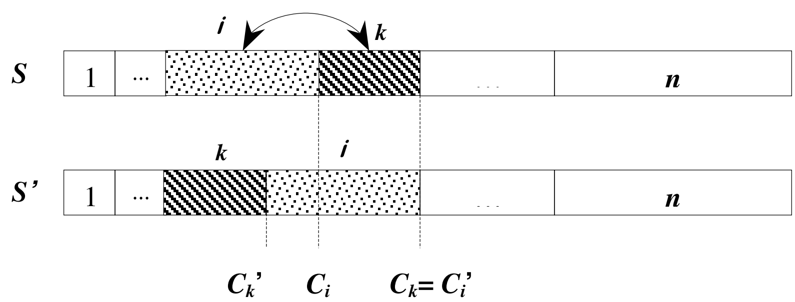
Assume that the jobs are numbered in non-decreasing order of their due dates d_j. In order to minimize the maximum lateness it is quite natural to process the jobs in this order. This dispatching rule is known as EDD (*Earliest Due Date*), sometimes it is called Jackson's rule due to R. Jackson who studied it in 1954.

We can prove

Theorem 1. For $1||L_{\max}$ the EDD-rule is optimal.

Proof (adjacent pairwise interchange argument)

Suppose a schedule *S*, which violates EDD, is optimal. In this schedule there must be at least two adjacent jobs *i* and *k* such that $d_i > d_k$ and job *i* precedes job *k*.



Swapping jobs *i* and *k* leads to a schedule *S'* such that

$$L_k' = C_k' - d_k < C_k - d_k$$
$$L_i' = C_i' - d_i = C_k - d_i < C_k - d_k$$
$$L_j' = C_j' - d_j = C_j - d_j = L_j \quad \text{for } j \neq i,k$$

which implies that $L_{\max}(S') = \max\left\{L_i', L_k', \max_{j\neq i,k}\{L_j'\}\right\} \le \max\left\{L_i, L_k, \max_{j\neq i,k}\{L_j\}\right\} = L_{\max}(S).$

If $L_{\max}(S') < L_{\max}(S)$, then we have obtained a contradiction to the assumption that *S* is optimal.

If $L_{\max}(S') = L_{\max}(S)$, then schedule *S* can be modified without increasing the objective function value so that jobs *i* and *k* do not violate *EDD*. ∎

EDD rule can be implemented in ……………….. time.

2. Minimising the number of late jobs: $1||\Sigma U_j$

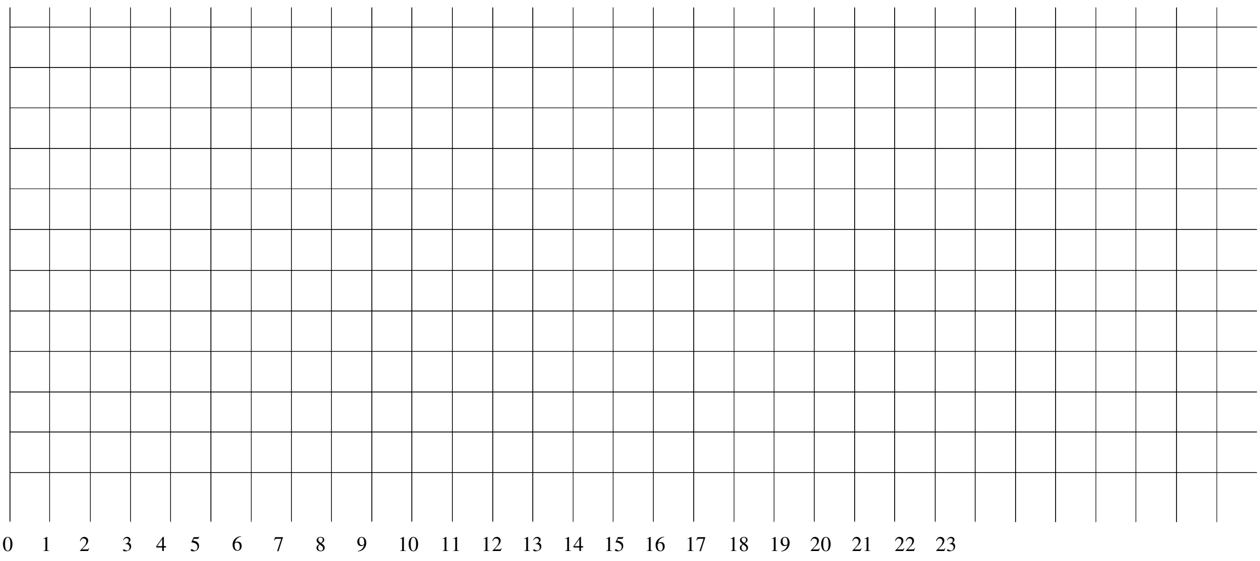
In this section we develop an algorithm that finds an optimal schedule for problem $\mathbf{1||\Sigma U_j}$, where U_j is a unit penalty for completing job j after its due date:

$$U_j = \begin{cases} 0, & \text{if } C_j \le d_j \\ 1, & \text{otherwise.} \end{cases}$$

Consider the following example.

Job	p_j	d_j
1	7	9
2	8	17
3	4	18
4	6	19
5	6	20

The jobs are in the EDD order. The first two jobs can be scheduled on-time while job 3 is late.
. . .



The optimal job sequence is (|), where the first ………… jobs are on-time and the last ………… jobs are late.

The value of the objective function for this schedule is

$$\Sigma U_j = \ldots\ldots\ldots$$

The algorithm that solves problem $\mathbf{1||\Sigma U_j}$ is known as Moore's algorithm, due to J.M. Moore who designed it in 1968.

Moore's algorithm.

- The algorithm repeatedly adds jobs in the EDD order to the end of a partial schedule of on-time jobs.
- If the addition of job j results in this job being completed after its due date d_j, then a job in the partial schedule with the largest processing time is removed and declared late.
- All late jobs are scheduled in an arbitrary order after on-time jobs.

The correctness of the algorithm can be proved by induction.
By removing the job with the largest processing time, we guarantee that the total processing time of on-time jobs is as small as possible.

The time complexity of Moore's algorithm is ……………………